Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

And at their intersections place
The Quakers's dwelling. For we find
These uprights are in each combined
Just two-thirds from the trigon-points
And one-third from the tangent-joints.
Each upright we can plainly see
In radius times square root of three; [$r\sqrt{3}$]
And root of three times radius squared [$r^2\sqrt{3}$]
Is trigon's area unimpaired.
A semicircle interjoined
With the Quaker's acre can be coined
An equal to the trigon's space. [$\frac{1}{2}\pi r^2+160$]
Now equal to each other place
The areas of the trigon found;
And if the work is true and sound,
We'll find the half of sixty-three [31.50+]
Is a trifle less than in rods should be
The radius of each circled bound
Wherein the sons their dwellings found.
Just twice the radius, or sixty-three, [63.0+]
As the rods apart the sons must be.
Two-thirds of the upright, as shown above,
The sons to their father will have to rove;
This distance, in rods, will two decimals run
In one-eighth of two hundred ninety-one. [36.37+]
Now we've told by skill and rhyming art
The number of rods they live apart.

II. Solution by J. M. HOWIE, State Normal School, Peru, Neb.; LESLIE J. LOCKE. M. A., Fredonia Institute, Fredonia, Pa.; O. S. WESTCOTT, Chicago, Ill.; J. W. DAPPERT. C. E., Taylorville, Ill.; B. L. REMICK, Bradley Institute, Peoria, Ill.; W. MANZILLA, Langston University, Langston, Okla.

Let ABC be the triangle formed by joining the centers of the farms, CE the altitude. Since the circles are equal, the triangle ACB is equilateral, and therefore $AC=AB=BC=2r$, where r is the radius of the equal circles.

Area of triangle $ABC=\frac{1}{2}AB\times CE=\frac{1}{2}.2r.\sqrt{(4r^2-r^2)}=r^2\sqrt{3}$.

The area of the three circular sectors included in the triangle$=3.\frac{1}{6}\pi r^2=\frac{1}{2}\pi r^2$.

$\therefore$ The area of the curvilinear triangle $EFI=\sqrt{3}r^2-\frac{1}{2}\pi r^2=160$ sq. rods.

$$\therefore r=\sqrt{\left(\frac{320}{2\sqrt{3}-\pi}\right)}=8\sqrt{\left(\frac{5}{2\sqrt{3}-\pi}\right)}=37.7 \text{ rods.}$$

$2r=AC=75.4$ rods=distance between sons' houses, and $\frac{2}{3}\sqrt{3}r=43.5323$ rods=distance from father's to sons' house.

Solved in a similar manner by *G. B. M. ZERR*, *J. SCHEFFER*, *C. C. CROSS*, *H. C. WHITAKER*, *ELMER SCHUYLER*, *ALOIS F. KOVARIK*, *JOHN T. FAIRCHILD*, *J. M. COLAW*, *HON. JOSIAH H. DRUMMOND*, *P. S. BERG*, *H. I. HOPKINS*, *COOPER D. SCHMITT*, and *J. O. MAHONEY*.

Solutions of problem 124 were received from *CHAS. C. CROSS*, *P. S. BERG*, *J. SCHEFFER*, *G. B. M. ZERR*, *ELMER SCHUYLER*, and *H. C. WHITAKER*.

ALGEBRA.

101. Proposed by G. B. M. ZERR, A.M., Ph.D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Prove that $(1+2+3+\dots+n)+\frac{n}{2\,!}(1^2+2^2+3^2+\dots+n^2)+\frac{n(n-1)}{3\,!}$

$$(1^3+2^3+3^3+\ldots.+n^3)+\frac{n(n-1)(n-2)}{4!}(1^4+2^4+3^4+\ldots.+n^4)+\ldots.$$

$$+\frac{n(n-1)(n-2)}{4!}(1^{n-3}+2^{n-3}+3^{n-3}+\ldots.+n^{n-3})+\frac{n(n-1)}{3!}(1^{n-2}+2^{n-2}+3^{n-2}$$

$$+\ldots.+n^{n-2})+\frac{n}{2!}(1^{n-1}+2^{n-1}+3^{n-1}+\ldots.+n^{n-1})+(1^n+2^n+3^n+\ldots.+n^n)$$

$=(n+1)^n-1$, and substitute for $n=2$, 3, 4, 5, and 6.

Solution by the PROPOSER.

$$(1+x)^{n+1}=1+(n+1)x+\frac{n(n+1)x^2}{2!}+\frac{n(n+1)(n-1)x^3}{3!}$$

$$+\frac{n(n+1)(n-1)(n-2)x^4}{4!}+\ldots.$$

$$\therefore\ \frac{(1+x)^{n+1}-1}{n+1}=x\left(1+\frac{nx}{2!}+\frac{n(n-1)x^2}{3!}+\frac{n(n-1)(n-2)x^3}{4!}+\ldots.\right).$$

Let $\frac{n}{2!}$, $\frac{n(n-1)}{3!}$, $\frac{n(n-1)(n-2)}{4!}$, etc.,$=a_1$, a_2, a_3, etc., respect.

$$\therefore\ \frac{(1+x)^{n+1}-1}{n+1}=x(1+a_1x+a_2x^2+a_3x^3+\ldots.+a_nx^n).$$

When $x=1$, $\frac{2^{n+1}-1}{n+1}=1+a_1+a_2+a_3+\ldots.+a_n$.

When $x=2$, $\frac{3^{n+1}-1}{n+1}=2+2^2a_1+2^3a_2+2^4a_3+\ldots.+2^{n+1}a_n$.

When $x=3$, $\frac{4^{n+1}-1}{n+1}=3+3^2a_1+3^3a_2+3^4a_3+\ldots.+3^{n+1}a_n$.

. . . .

When $x=n$, $\frac{(n+1)^{n+1}-1}{n+1}=n+n^2a_1+n^3a_2+n^4a_3+\ldots.n^{n+1}a_n$

Adding, we get at once,

$$\frac{2^{n+1}+3^{n+1}+4^{n+1}+\ldots.+(n+1)^{n+1}-n}{(n+1)}$$

$$=(1+2+3+4+\ldots+n)+(1^2+2^2+3^2+4^2+\ldots.+n^2)a_1+(1^3+2^3+3^3+4^3$$

$$+\ldots.+n^3)a_2+\ldots.+(1^n+2^n+3^n+4^n+\ldots.4^n+\ldots.+n^n)a_{n-1}$$

$+(1n^{n+1}+2^{n+1}+3^{n+1}+4^{n+1}+\ldots\ldots+n^{n+1})a_n.$

Transposing the last quantity in parenthesis, reducing, and replacing the values of a_1, a_2, a_3, we get,

$$(1+2+3+\ldots\ldots+n)+\frac{n}{2\,!}(1^2+2^2+3^2+\ldots\ldots+n^2)+\frac{n(n-1)}{3\,!}(1^3+2^3+3^3+\ldots n^3)$$

$$+\ldots\ldots+\frac{n(n-1)}{3\,!}(1^{n-2}+2^{n-2}+3^{n-2}+\ldots\ldots+n^{n-2})+\frac{n}{2\,!}(1^{n-1}+2^{n-1}+3^{n-1}$$

$$+\ldots\ldots+n^{n-1})+(1^n+2^n+3^n+\ldots\ldots+n^n)=(n+1)^n-1.$$

When $n=2$, we get $(1+2)+(1^2+2^2)=3^2-1=8$.

When $n=3$, $(1+2+3)+\frac{3}{2}(1^2+2^2+3^2)+(1^3+2^3+3^3)=4^3-1=63$.

When $n=4$, $(1+2+3+4)+2(1^2+2^2+3^2+4^2)+2(1^3+2^3+3^3+4^3)$ $+(1^4+2^4+3^4+4^4)=5^4-1=624$.

When $n=5$, $(1+2+3+4+5)+\frac{5}{2}(1^2+2^2+3^2+4^2+5^2)+\frac{10}{3}(1^3+2^3+3^3$ $+4^3+5^3)+\frac{5}{2}(1^4+2^4+3^4+4^4+5^4)+(1^5+2^5+3^5+4^5+5^5)=6^5-1=7775$.

When $n=6$, $(1+2+3+4+5+6)+3(1^2+2^2+3^2+4^2+5^2+6^2)+5(1^3+2^3+3^3$ $+4^3+5^3+6^3)+5(1^4+2^4+3^4+4^4+5^4+6^4)+3(1^5+2^5+3^5+4^5+5^5+6^5)+(1^6+2^6+3^6$ $+4^6+5^6+6^6)=7^6-1=117648$.

Also solved by *ELMER SCHUYLER*.

102. Proposed by J. MARCUS BOORMAN, Woodmere, N. Y.

Solve $2x+\sqrt{[x^2-7]}=5$.

I. Solution by COOPER D. SCHMITT, A. M., University of Tennessee, Knoxville, Tenn.; M. A. GRUBER, A. M., Washington, D. C.; J. SCHEFFER, A. M., Hagerstown, Md; and G. B. M. ZERR, A. M., Ph. D., Chester High School, Chester, Pa.

Transposing, $\sqrt{(x^2-7)}=5-2x$.

Squaring, $x^2-7=25-20x+4x^2$; whence $3x^2-20x+32=0$.

Solving, $x=4$ or $\frac{8}{3}$.

Neither of these satisfies the original equation, but by writing it thus, $2x-\sqrt{(x^2-7)}=5$, both values will satisfy it.

II. Solution by H. C. WHITAKER, M. E., Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pa.

Denote $2x-5$ by y; then the given equation reduces to

$$+\sqrt{(y^2+10y-3)}=-2y.$$

Divide by y, $\qquad +\sqrt{(1+10/y-3/y^2)}=-2\ldots\ldots(1).$

But this equation is absurd, since it makes a positive square root equal to a negative number.